

A COSMOLOGY-INDEPENDENT CALIBRATION OF GAMMA-RAY BURST LUMINOSITY RELATIONS AND THE HUBBLE DIAGRAM

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ABSTRACT

An important concern in the application of gamma-ray bursts (GRBs) to cosmology is that the calibration of GRB luminosity/energy relations depends on the cosmological model, due to the lack of a sufficient low-redshift GRB sample. In this paper, we present a new method to calibrate GRB relations in a cosmology-independent way. Since objects at the same redshift should have the same luminosity distance and since the distance moduli of Type Ia supernovae (SNe Ia) obtained directly from observations are completely cosmology independent, we obtain the distance modulus of a GRB at a given redshift by interpolating from the Hubble diagram of SNe Ia. Then we calibrate seven GRB relations without assuming a particular cosmological model and construct a GRB Hubble diagram to constrain cosmological parameters. From the 42 GRBs at $1.4 < z \leq 6.6$, we obtain $\Omega_M = 0.25^{+0.04}_{-0.05}$, $\Omega_\Lambda = 0.75^{+0.05}_{-0.04}$ for the flat Λ CDM model, and for the dark energy model with a constant equation of state $w_0 = -1.05^{+0.27}_{-0.40}$, which is consistent with the concordance model in a 1- σ confidence region.

Subject headings: gamma rays: bursts — cosmology: observations

1. INTRODUCTION

In the past decade, observations of Type Ia supernovae (SNe Ia; Riess et al. 1998; Schmidt et al. 1998; Perlmutter et al. 1999), cosmic microwave background (CMB) fluctuations (Bennett et al. 2003; Spergel et al. 2003, 2007), and large-scale structures (LSS; Tegmark et al. 2004, 2006) have been used to explore cosmology extensively. These observations are found to be consistent with the so-called concordance cosmology, in which the universe is spatially flat and contains pressureless matter and dark energy, with fractional energy densities of $\Omega_M = 0.27 \pm 0.04$ and $\Omega_\Lambda = 0.73 \pm 0.04$ (Davis et al. 2007). Observations of SN Ia provide a powerful probe in the modern cosmology. Phillips (1993) found that there is an intrinsic relation between the peak luminosity and the shape of the light curve of SNe Ia. This relation and other similar luminosity relations make SNe Ia standard candles for measuring the geometry and dynamics of the universe. However, the maximum redshift of the SNe Ia which we can currently use is only about 1.7, whereas fluctuations of the CMB provide the cosmological information from last scattering surface at $z = 1089$. Therefore, the earlier universe at higher redshift may not be well-studied without data from standardized candles in the “cosmological desert” from the SNe Ia redshift limit to $z \sim 1000$.

Gamma-Ray Bursts (GRBs) are the most intense explosions observed so far. Their high energy photons in the gamma-ray band are almost immune to dust extinction, whereas in the case of SN Ia observations, there is extinction from the interstellar medium when optical photons propagate towards us. Moreover, GRBs are likely to occur in the high-redshift range up to at least $z = 6.6$ (Krimm et al. 2006); higher redshift GRBs up to $z = 10$ should have already been detected,

although none have been identified (Lamb & Reichart 2000; Bromm & Loeb 2002, 2006; Lin, Zhang, & Li 2004). Thus, by using GRBs, we may explore the early universe in the high redshift range which is difficult to access by other cosmological probes. These advantages make GRBs attractive for cosmology research.

GRB luminosity/energy relations are connections between measurable properties of the prompt gamma-ray emission and the luminosity or energy. In recent years, several empirical GRB luminosity relations have been proposed as distance indicators (see e.g. Ghirlanda et al. 2006a; Schaefer (2007) for reviews), such as the luminosity-spectral lag (L - τ_{lag}) relation (Norris, Marani, & Bonnell 2000), the luminosity-variability (L - V) relation (Fenimore & Ramirez-Ruiz 2000; Reichart et al. 2001), the isotropic energy-peak spectral energy (E_{iso} - E_p) relation (i.e., the so-called Amati relation, Amati et al. 2002), the collimation-corrected energy-peak spectral energy (E_γ - E_p) relation (i.e., the so-called Ghirlanda et al. 2004a), the L - E_p relation (Schaefer 2003a; Yonetoku et al. 2004), and two multi-variable relations. The first of these multiple relations is between E_{iso} , E_p and the break time of the optical afterglow light curves (t_b) (i.e., the so-called Liang-Zhang relation, Liang & Zhang 2005); the other is between luminosity, E_p and the rest-frame “high-signal” timescale ($T_{0.45}$) (Firmani et al. 2006).

Several authors have made use of these GRB luminosity indicators as standard candles at very high redshift for cosmology research (e.g. Schaefer 2003b; Dai et al. 2004; Ghirlanda et al. 2004b; Firmani et al. 2005, 2006, 2007; Liang & Zhang 2005; Xu et al. 2005; Wang & Dai 2006; see also e.g. Ghirlanda et al. 2006a and Schaefer (2007) for reviews). Schaefer (2003b) derived the luminosity distances of nine GRBs with known redshifts by using two GRB luminosity relations to construct the first GRB Hubble diagram. Dai et al. (2004) considered the Ghirlanda relation with 12 bursts and proposed another approach to constrain cosmological parameters. Liang & Zhang (2005) constrained cosmological parameters and the transition redshift using the E_{iso} - E_p - t_b relation. More recently, Schaefer (2007) used five GRB relations calibrated with 69 GRBs by assuming two adopted

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cosmological models to obtain the derived distance moduli for plotting the Hubble diagram, and joint constraints on the cosmological parameters and dark energy models have been derived in many works by combining the 69 GRB data with SNe Ia and the other cosmological probes, such as the CMB anisotropy, the baryon acoustic oscillation (BAO) peak, the X-ray gas mass fraction in clusters, the linear growth rate of perturbations, and the angular diameter distances to radio galaxies (Wright 2007; Wang et al. 2007; Li et al. 2008; Qi et al. 2008; Daly et al. 2008).

However, an important point related to the use of GRBs for cosmology is the dependence on the cosmological model in the calibration of GRB relations. In the case of SN Ia cosmology, the calibration is carried out with a sample of SNe Ia at very low redshift where the luminosities of SNe Ia are essentially independent of any cosmological model (i.e., at $z \leq 0.1$, the luminosity distance has a negligible dependence on the choice of the model). However, in the case of GRBs, the observed long-GRB rate falls off rapidly at low redshifts, and some nearby GRBs may be intrinsically different (e.g., GRB 980425, GRB 031203; Norris 2002; Soderberg et al. 2004; Guetta et al. 2004; Liang & Zhang 2006a). Therefore, it is very difficult to calibrate the relations with a low-redshift sample. The relations of GRBs presented above have been calibrated by assuming a particular cosmological model (e.g. the Λ CDM model). In order to investigate cosmology, the relations of standard candles should be calibrated in a cosmological model-independent way. Otherwise, the circularity problem cannot easily be avoided. Many of the works mentioned above treat the circularity problem with a statistical approach. A simultaneous fit of the parameters in the calibration curves and the cosmology is carried out to find the optimal GRB relation and the optimal cosmological model in the sense of a minimum scattering in both the luminosity relations and the Hubble diagram. Firmani et al. (2005) has also proposed a Bayesian method to get around the circularity problem. Li et al. (2008b) presented another new method to deal with the problem, using Markov chain Monte Carlo (MCMC) global fitting analysis. Instead of a hybrid sample over the whole redshift range of GRBs, Takahashi et al. (2003) first calibrated two GRB relations at low redshift ($z \leq 1$), where distance-redshift relations have been already determined from SN Ia; they then used GRBs at high redshift ($z > 1$) as a distance indicator. Bertolami & Silva (2006) considered the use of GRBs at $1.5 < z < 5$ calibrated with the bursts at $z \leq 1.5$ as distance markers to study the unification of dark energy and dark matter in the context of the generalized Chaplygin gas model. Schaefer (2007) made a detailed comparison between the Hubble diagram calibrated only with 37 bursts at $z < 2$ and the one calibrated with the data from all 69 GRBs to show that the results from fits to the GRB Hubble diagram calibrated with the hybrid sample are robust. These calibration methods above carried out with the sample at low redshifts have been derived from the Λ CDM model. However, we note that the circularity problem can not be circumvented completely by means of statistical approaches, because a particular cosmology model is required in doing the joint fitting. This means that the parameters of the calibrated relations are still coupled to the cosmological parameters derived from a given cosmological model. In principle, the circularity problem can be avoided in two ways (Ghirlanda et al. 2006a): (1) through a solid physical interpretation of these relations that would fix their slope independently from cosmological model, or (2) through the calibration of these relations by sev-

eral low-redshift GRBs. Recently the possibility of calibrating the standard candles using GRBs in a low-dispersion in redshift near a fiducial redshift has been proposed (Lamb et al. 2005; Ghirlanda et al. 2006b). Liang & Zhang (2006b) elaborated this method further based on Bayesian theory. However, the GRB sample available now is far from what is needed to calibrate the relations in this way.

As analyzed above, due to the lack of low-redshift GRBs, the methods used in GRB luminosity relations for cosmology are different from the standard Hubble diagram method used in SN Ia cosmology. In this work, we present a new method to calibrate the GRB relations in a cosmological model-independent way. In the case of SN Ia cosmology, the distance of nearby SNe Ia used to calibrate the luminosity relations can be obtained by measuring Cepheid variables in the same galaxy, and with other distance indicators. Thus, Cepheid variables have been regarded as the first-order standard candles for calibrating SNe Ia, the second-order standard candles. It is obvious that objects at the same redshift should have the same luminosity distance in any cosmology. For calibrating GRBs, there are so many SNe Ia (e.g., 192 SNe Ia used in Davis et al. 2007) that we can obtain the distance moduli (also the luminosity distance) at any redshift in the redshift range of SNe Ia by interpolating from SN Ia data in the Hubble diagram. Furthermore, the distance moduli of SNe Ia obtained directly from observations are completely cosmological model independent. Therefore, in the same sense as Cepheid variables and SNe Ia, if we regard SNe Ia as the first order standard candles, we can obtain the distance moduli of GRBs in the redshift range of SNe Ia and calibrate GRB relations in a completely cosmological model-independent way. Then if we further assume that these calibrated GRB relations are still valid in the whole redshift range of GRBs, just as for SNe Ia, we can use the standard Hubble diagram method to constrain the cosmological parameters from the GRB data at high redshift obtained by utilizing the relations.

The structure of this paper is arranged as follow. In section 2, we calibrate seven GRB luminosity/energy relations with the sample at $z \leq 1.4$ obtained by interpolating from SNe Ia data in the Hubble diagram. In section 3, we construct the Hubble diagram of GRBs obtained by using the interpolation methods and constrain cosmological parameters. Conclusions and a discussion are given in section 4.

2. THE CALIBRATION OF THE LUMINOSITY RELATIONS OF GAMMA-RAY BURSTS

We adopt the data for 192 SNe Ia (Riess et al. 2007; Wood-Vasey et al. 2007; Astier et al. 2007, Davis et al. 2007), shown in Figure 1. It is clear that for GRBs in the redshift range of SNe Ia, there are enough data SN Ia points and the redshift intervals of the neighboring SN Ia data points are also small enough, to be able to use interpolation methods to obtain the distance moduli of GRBs. Since there is only one SN Ia point at $z > 1.4$ (the redshift of SN1997ff is $z = 1.755$), we initially exclude it from our SN Ia sample used to interpolate the distance moduli of GRBs in the redshift range of the SN Ia sample (191 SNe Ia data at $z \leq 1.4$). As the distance moduli of SNe Ia data in the Hubble diagram are obtained directly from observations, with the sample at $z \leq 1.4$ by interpolating from the Hubble diagram of SNe Ia, we can therefore calibrate GRB luminosity relations in a completely cosmology-independent way.

In this section, we calibrate seven GRB luminosity/energy relations with the sample at $z \leq 1.4$, i.e., the $\tau_{\text{lag}}-L$ relation, the

V - L relation, the L - E_p relation, the E_γ - E_p relation, the τ_{RT} - L relation, where τ_{RT} is the minimum rise time in the GRB light curve (Schaefer, 2007), the E_{iso} - E_p relation, and the E_{iso} - E_p - t_b relation.

The isotropic luminosity of a burst is calculated by

$$L = 4\pi d_L^2 P_{\text{bolo}}, \quad (1)$$

where d_L is the luminosity distance of the burst and P_{bolo} is the bolometric flux of gamma-rays in the burst. The isotropic energy released from a burst is given by

$$E_{\text{iso}} = 4\pi d_L^2 S_{\text{bolo}}(1+z)^{-1}, \quad (2)$$

where S_{bolo} is the bolometric fluence of gamma-rays in the burst at redshift z . The total collimation-corrected energy is then calculated by

$$E_\gamma = F_{\text{beam}} E_{\text{iso}}, \quad (3)$$

where the beaming factor, F_{beam} is $(1 - \cos\theta_{\text{jet}})$ with the jet opening angle (θ_{jet}), which is related to the break time (t_b).

A GRB luminosity relation can be generally written in the form

$$\log y = a + b \log x, \quad (4)$$

where a and b are the intercept and slope of the relation respectively; y is the luminosity (L in units of erg s^{-1}) or energy (E_{iso} or E_γ in units of erg); x is the GRB parameters measured in the rest frame, e.g., $\tau_{\text{lag}}(1+z)^{-1}/(0.1 \text{ s})$, $V(1+z)/0.02$, $E_p(1+z)/(300 \text{ keV})$, $\tau_{RT}(1+z)^{-1}/(0.1 \text{ s})$, $E_p(1+z)/(300 \text{ keV})$, for the 6 two-variable relations above. We adopt the data for these quantities from Schaefer (2007). For the one multi-variable relation (i.e., E_{iso} - E_p - t_b), the calibration equation is

$$\log y = a + b_1 \log x_1 + b_2 \log x_2, \quad (5)$$

where x_1 and x_2 are $E_p(1+z)/(300 \text{ keV})$, $t_b/(1+z)/(1 \text{ day})$ respectively, and b_1 and b_2 are the slopes of x_1 and x_2 respectively. The error of the interpolated distance modulus of a GRB must account for the original uncertainties of the SNe Ia as well as for the uncertainties from the interpolation. When the linear interpolation is used, the error can be calculated by

$$\sigma_\mu = [((z_{i+1}-z)/(z_{i+1}-z_i))^2 \epsilon_{\mu,i}^2 + ((z-z_i)/(z_{i+1}-z_i))^2 \epsilon_{\mu,i+1}^2]^{1/2}, \quad (6)$$

where σ_μ is the error of the interpolated distance modulus, μ is the interpolated distance modulus of a source at redshift z , $\epsilon_{\mu,i}$ and $\epsilon_{\mu,i+1}$ are errors of the SNe, μ_i and μ_{i+1} are the distance moduli of the SNe at nearby redshifts z_i and z_{i+1} respectively. When the cubic interpolation is used, the error can be calculated by

$$\sigma_\mu = (A_0^2 \epsilon_{\mu,i}^2 + A_1^2 \epsilon_{\mu,i+1}^2 + A_2^2 \epsilon_{\mu,i+2}^2 + A_3^2 \epsilon_{\mu,i+3}^2)^{1/2}, \quad (7)$$

where $\epsilon_{\mu,i}$, $\epsilon_{\mu,i+1}$, $\epsilon_{\mu,i+2}$, and $\epsilon_{\mu,i+3}$ are errors of the SNe; and μ_i , μ_{i+1} , μ_{i+2} , and μ_{i+3} are the distance moduli of the SNe at nearby redshifts z_i , z_{i+1} , z_{i+2} , and z_{i+3} :

$$\begin{aligned} A_0 &= [(z_{i+1}-z)(z_{i+2}-z)(z_{i+3}-z)]/[(z_{i+1}-z_i)(z_{i+2}-z_i)(z_{i+3}-z_i)]; \\ A_1 &= [(z_i-z)(z_{i+2}-z)(z_{i+3}-z)]/[(z_i-z_{i+1})(z_{i+2}-z_{i+1})(z_{i+3}-z_{i+1})]; \\ A_2 &= [(z_i-z)(z_{i+1}-z)(z_{i+3}-z)]/[(z_i-z_{i+2})(z_{i+1}-z_{i+2})(z_{i+3}-z_{i+2})]; \\ A_3 &= [(z_i-z)(z_{i+1}-z)(z_{i+2}-z)]/[(z_i-z_{i+3})(z_{i+1}-z_{i+3})(z_{i+2}-z_{i+3})]. \end{aligned}$$

We determine the values of the intercept (a) and the slope (b) with their $1-\sigma$ uncertainties calibrated with the GRB sample at $z \leq 1.4$ by using two interpolation methods (the linear interpolation methods and the cubic interpolation method). For the 6 two-variable relations we use the same method (the bisector of the two ordinary least-squares) as used in Schaefer (2007), and for the one multi-variable relation, the multiple variable regression analysis is used. The bisector of the

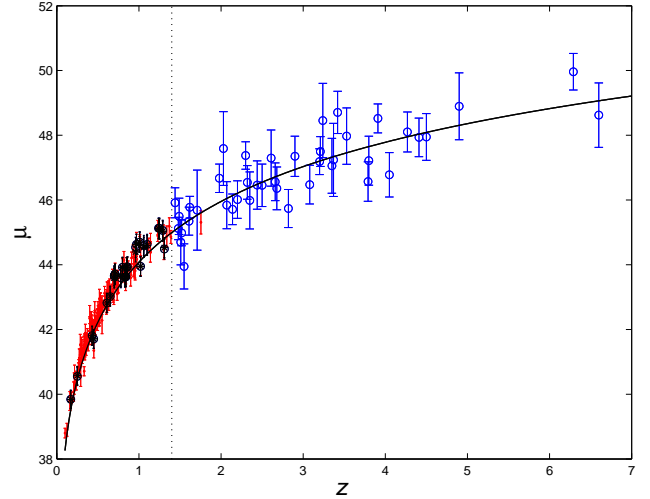


FIG. 1.— Hubble Diagram of 192 SNe Ia (red dots) and the 69 GRBs (circles) obtained using the interpolation methods. The 27 GRBs at $z \leq 1.4$ are obtained by interpolating from SN Ia data (black circles: the cubic interpolation method; black stars: the linear interpolation method); and the 42 GRBs at $z > 1.4$ (blue circles) are obtained with the five relations calibrated with the sample at $z \leq 1.4$ using the cubic interpolation method. The curve is the theoretical distance modulus in the concordance model ($w_0 = -1$, $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$), and the vertical dotted line represents $z = 1.4$.

two ordinary least-squares (Isobe et al., 1990) does not take the errors into account; but the use of weighted least-squares, taking into account the measurable uncertainties, results in almost identical best fits. However, taking into account the measurable uncertainties in the regression, when they are smaller than the intrinsic error, indeed will not change the fitting parameters significantly (for further discussion, see Schaefer 2007). The calibration results are summarized in Table 1, and we plot the GRB data at $z \leq 1.4$ with the distance moduli obtained by using the two interpolation methods from SN Ia data in Figure 1 for comparison. In the previous treatment, the distance moduli of GRBs are obtained by assuming a particular cosmological model with a hybrid sample in the whole redshift range of GRBs to calibrate the relations. Therefore, for comparison in Table 1 we also list the results calibrated with the same sample ($z \leq 1.4$) assuming the Λ CDM model ($w = -1$) or the Riess cosmology⁴ ($w = -1.31 + 1.48z$, Riess et al. 2004), which were used to calibrate the five GRB relations in Schaefer (2007). The calibration results obtained by using the interpolation methods are carried out with the 27 GRBs at $z \leq 1.4$, namely, 13, 19, 25, 12, 24, 12, and 12 GRBs for the τ_{lag} - L , V - L , L - E_p , E_γ - E_p , τ_{RT} - L , E_{iso} - E_p , and E_{iso} - E_p - t_b relations respectively;⁵ therefore, the uncertainties of the results are somewhat larger than those with the hybrid GRB sample.⁶

⁴ Here we follow Schaefer (2007) in calling this particular parametrization of the equation of state the "Riess cosmology".

⁵ For the seven relations, only data from Table 4 in Schaefer (2007) are used for the calibration. However, we note that there are more GRB data available to calibrate the E_{iso} - E_p relation.

⁶ Calibration results with their $1-\sigma$ uncertainty for the five GRB relations with 69 GRBs ($0.1 < z \leq 6.6$) obtained by assuming the Λ CDM model in Schaefer (2007) are also given in Table 1, for comparison.

TABLE 1
CALIBRATION RESULTS

Relation	Linear interpolation method		Cubic interpolation method		The Λ CDM model		The Riess cosmology		Schaefer (2007)	
	a	b	a	b	a	b	a	b	a	b
$\tau_{\text{lag}}-L$	52.22 ± 0.09	-1.07 ± 0.14	52.22 ± 0.09	-1.07 ± 0.13	52.15 ± 0.10	-1.11 ± 0.14	52.13 ± 0.09	-1.10 ± 0.13	52.26 ± 0.06	-1.01 ± 0.05
$V-L$	52.58 ± 0.13	2.04 ± 0.26	52.59 ± 0.13	2.05 ± 0.27	52.48 ± 0.13	2.04 ± 0.30	52.47 ± 0.13	2.02 ± 0.30	52.49 ± 0.22	1.77 ± 0.12
$L-E_p$	52.25 ± 0.09	1.68 ± 0.11	52.26 ± 0.09	1.69 ± 0.11	52.15 ± 0.09	1.66 ± 0.12	52.15 ± 0.09	1.64 ± 0.12	52.21 ± 0.13	1.68 ± 0.05
$E_\gamma-E_p$	50.70 ± 0.06	1.77 ± 0.17	50.71 ± 0.07	1.79 ± 0.18	50.59 ± 0.06	1.75 ± 0.19	50.59 ± 0.06	1.73 ± 0.19	50.57 ± 0.09	1.63 ± 0.03
$\tau_{\text{RT}}-L$	52.63 ± 0.11	-1.31 ± 0.15	52.64 ± 0.11	-1.31 ± 0.15	52.52 ± 0.10	-1.30 ± 0.15	52.51 ± 0.10	-1.29 ± 0.15	52.54 ± 0.06	-1.21 ± 0.06
$E_{\text{iso}}-E_p$	52.99 ± 0.16	1.88 ± 0.23	53.00 ± 0.17	1.90 ± 0.23	52.88 ± 0.16	1.84 ± 0.21	52.88 ± 0.15	1.81 ± 0.20		
$E_{\text{iso}}-E_p-t_b$ ^a	52.83 ± 0.10	2.26 ± 0.30	52.83 ± 0.10	2.28 ± 0.30	52.73 ± 0.10	2.19 ± 0.30	52.73 ± 0.10	2.16 ± 0.30		
		-1.07 ± 0.21		-1.07 ± 0.21		-1.01 ± 0.20		-0.99 ± 0.21		

NOTE. — Calibration results (for a =intercept, b =slope) with their $1-\sigma$ uncertainties, for the seven GRB luminosity/energy relations with the sample at $z \leq 1.4$, using two interpolation methods (the linear interpolation method and the cubic interpolation methods) directly from SNe Ia data and by assuming a particular cosmological models (the Λ CDM model or the Riess cosmology). Only the data available from Table 4 in Schaefer 2007 are used to calibrate the seven GRB relations. For comparison, calibration results with the 69 GRBs obtained by assuming the Λ CDM model for the five GRB relations in Schaefer 2007 are given in the last column.

^aIn the slope values for the $E_{\text{iso}}-E_p-t_b$ relation, the first line is b_1 , and the second line is b_2 .

The linear correlation coefficients of the calibration with the sample at $z \leq 1.4$ using the cubic interpolation methods are -0.88, 0.65, 0.89, 0.94, -0.75, and 0.68 for the above 6 two-variable relations respectively, and 0.94 for E_{iso} vs. the combined variable $(b_1 \log[E_p(1+z)] + b_2 \log[t_b/(1+z)])$ for the $E_{\text{iso}}-E_p-t_b$ relation, which shows that these correlations are significant.

From Table 1 and Figure 1, we find that the calibration results obtained using the linear interpolation methods are almost identical to the results calibrated by using the cubic interpolation method. We also find the results obtained by assuming the two cosmological models with the same sample differ only slightly from, but are still fully consistent with, those calibrated using our interpolation methods. The reason for this is easy to understand, since both cosmological models are fully compatible with SN Ia data. Nevertheless, it should be noticed that the calibration results obtained using the interpolation methods directly from SN Ia data are completely cosmology independent.

3. THE HUBBLE DIAGRAM OF GAMMA-RAY BURSTS

If we further assume GRB luminosity/energy relations do not evolve with redshift, we are able to obtain the luminosity (L) or energy (E_{iso} or E_γ) of each burst at high redshift ($z > 1.4$) by utilizing the calibrated relations. Therefore, the luminosity distance (d_L) can be derived from equation (1~3). The uncertainty of the value of the luminosity or energy deduced from a GRB relation is

$$\sigma_{\log y}^2 = \sigma_a^2 + (\sigma_b \log x)^2 + (0.4343 b \sigma_x / x)^2 + \sigma_{\text{sys}}^2, \quad (8)$$

where σ_a , σ_b and σ_x are 1- σ uncertainty of the intercept, the slope and the GRB measurable parameters, and σ_{sys} is the systematic error in the fitting that accounts for the extra scatter of the luminosity relations. The value of σ_{sys} can be estimated by finding the value such that a χ^2 fit to the calibration curve produces a value of reduced χ^2 of unity (Schaefer 2007). A distance modulus can be calculated as $\mu = 5 \log d_L + 25$ (where d_L is in Mpc). The propagated uncertainties will depend on whether P_{bolo} or S_{bolo} is used:

$$\sigma_\mu = [(2.5 \sigma_{\log L})^2 + (1.086 \sigma_{P_{\text{bolo}}} / P_{\text{bolo}})^2]^{1/2}, \quad (9)$$

or

$$\sigma_\mu = [(2.5 \sigma_{\log E_{\text{iso}}})^2 + (1.086 \sigma_{S_{\text{bolo}}} / S_{\text{bolo}})^2]^{1/2}, \quad (10)$$

and

$$\sigma_\mu = [(2.5 \sigma_{\log E_\gamma})^2 + (1.086 \sigma_{S_{\text{bolo}}} / S_{\text{bolo}})^2 + (1.086 \sigma_{F_{\text{beam}}} / F_{\text{beam}})^2]^{1/2}. \quad (11)$$

We use the same method as used in Schaefer (2007) to obtain the best estimate μ for each GRB which is the weighted average of all available distance moduli. The derived distance modulus for each GRB is

$$\mu = (\sum_i \mu_i / \sigma_{\mu_i}^2) / (\sum_i \sigma_{\mu_i}^{-2}), \quad (12)$$

with its uncertainty $\sigma_\mu = (\sum_i \sigma_{\mu_i}^{-2})^{-1/2}$, where the summations run from 1 to 5 over the five relations used in Schaefer (2007) with available data.

We have plotted the Hubble diagram of the 69 GRBs obtained using the interpolation methods in Figure 1. The 27 GRBs at $z \leq 1.4$ are obtained by using two interpolation methods directly from SNe data. The 42 GRB data at $z > 1.4$ are obtained by utilizing the five relations calibrated with the sample at $z \leq 1.4$ using the cubic interpolation method.

Then the cosmological parameters can be fitted by the minimum χ^2 method. The definition of χ^2 is

$$\chi^2 = \sum_{i=1}^N \frac{(\mu_{\text{th},i} - \mu_{\text{obs},i})^2}{\sigma_{\mu_i}^2}, \quad (13)$$

in which $\mu_{\text{th},i}$ is the theoretical value of distance modulus, $\mu_{\text{obs},i}$ is the observed distance modulus with its error σ_{μ_i} . The theoretical value of the distance modulus (μ_{th}) depends on the theoretical value of luminosity distance. For the Λ CDM model, the luminosity distance is calculated by

$$d_L = \frac{c(1+z)}{H_0 \sqrt{|\Omega_k|}} \times \text{sinn}[\sqrt{|\Omega_k|} \int_0^z \frac{dz}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\Lambda}}], \quad (14)$$

where $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$, and $\text{sinn}(x)$ is $\sinh(x)$ for $\Omega_k > 0$, \sin for $\Omega_k < 0$, and x for $\Omega_k = 0$. For the dark energy model with a constant equation of state (w_0), the luminosity distance in a flat universe is

$$d_L = \frac{c(1+z)}{H_0} \times \int_0^z \frac{dz}{\sqrt{(1+z)^3 \Omega_m + (1-\Omega_m)(1+z)^{3(1+w_0)}}}, \quad (15)$$

here we adopt $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Figure 2a shows the joint confidence regions for $(\Omega_M, \Omega_\Lambda)$ in the Λ CDM model from the 42 GRB data ($z > 1.4$) obtained by utilizing the five relations calibrated with the sample at $z \leq 1.4$ using the cubic interpolation method. The 1- σ confidence region is $(\Omega_M, \Omega_\Lambda) = (0.25_{-0.08}^{+0.07}, 0.68_{-0.64}^{+0.37})$ with $\chi_{\text{min}}^2 = 44.40$ for 40 degrees of freedom. For a flat universe, we obtain $\Omega_M = 0.25_{-0.05}^{+0.04}$ and $\Omega_\Lambda = 0.75_{-0.04}^{+0.05}$, which is consistent with the concordance model ($\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$) in the 1- σ confidence region. From equation (14), it is obvious that for the Λ CDM model, the theoretical value of the luminosity distance mainly depends on Ω_M at higher redshift, and strongly depends on Ω_Λ at lower redshift. Therefore, we can find that the shape of the likelihood contour of GRBs at higher redshift is almost vertical to the horizontal axis (Ω_M) compared to that of SNe Ia at lower redshift. To understand the shape of the confidence regions in the $(\Omega_M, \Omega_\Lambda)$ plane, Firmani et al. (2007) explored the behavior of the luminosity distance d_L at different redshifts z in a given cosmological parameter space. The stripe where d_L varies by 1% at $z = 3$, shown in the $(\Omega_M, \Omega_\Lambda)$ plane, was almost vertical to the Ω_M axis, which corresponds roughly to the typical redshifts ($z \sim 3$) of the GRB sample at $z > 1.4$. If we add the one SN Ia point (SN1997ff at $z = 1.755$) at $z > 1.4$ into our SN Ia sample used to interpolate the distance moduli of GRBs, we can calibrate the GRB relations with 36 GRBs at $z \leq 1.76$ by using the cubic interpolation method. We find that the fitting results from the 33 GRB data ($z > 1.76$) obtained by utilizing the relations calibrated with the sample at $z \leq 1.76$ using the cubic interpolation method is $\Omega_M = 0.28_{-0.07}^{+0.06}$ for a flat universe (Figure 2b), which is consistent with the result from the 42 GRB data ($z > 1.4$) obtained using the cubic interpolation method. However, if the relations are calibrated with the GRB sample at $z \leq 1.4$ by assuming a particular cosmological model, we find that the fitting results the 42 GRB data ($z > 1.4$) obtained assuming the Λ CDM model or the Riess cosmology are $\Omega_M = 0.38_{-0.07}^{+0.07}$ (Figure 2c) or $\Omega_M = 0.39_{-0.07}^{+0.07}$ (Figure 2d) for a flat universe, which are systematically higher than $\Omega_M = 0.27_{-0.04}^{+0.04}$ beyond a 1- σ deviation.

Figure 3 shows likelihood contours in the (Ω_M, w_0) plane in the dark energy model with a constant w_0 for a flat universe

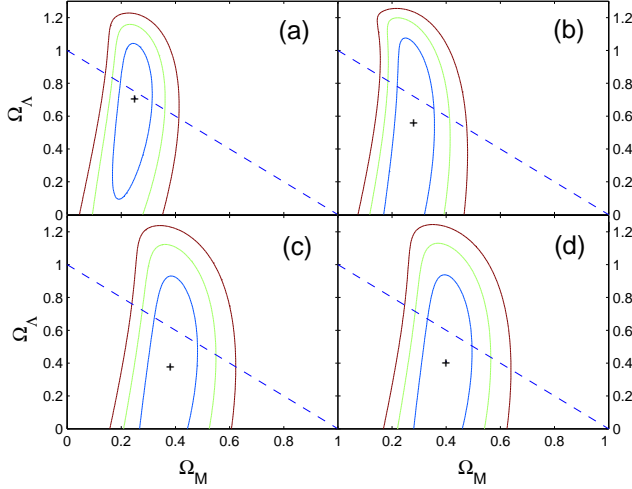


FIG. 2.— (a) Joint confidence regions for $(\Omega_M, \Omega_\Lambda)$ in the Λ CDM model from the data for 42 GRBs ($z > 1.4$) obtained by utilizing the five relations calibrated with the sample at $z \leq 1.4$ using the cubic interpolation method. The plus sign indicates the best fit values. The contours correspond to 1, 2, and 3- σ confidence regions, and the dashed line represents the flat universe. The 1- σ confidence region is $(\Omega_M, \Omega_\Lambda) = (0.25^{+0.07}_{-0.08}, 0.68^{+0.37}_{-0.64})$. For a flat universe prior, $\Omega_M = 0.25^{+0.04}_{-0.05}$ and $\Omega_\Lambda = 0.75^{+0.05}_{-0.04}$. (b) Joint confidence regions for $(\Omega_M, \Omega_\Lambda)$ from the data for 33 GRBs ($z > 1.76$) obtained by utilizing the relations calibrated with the sample at $z \leq 1.76$ by using the cubic interpolation method. For a flat universe prior, $\Omega_M = 0.28^{+0.06}_{-0.07}$. (c) The joint confidence regions for $(\Omega_M, \Omega_\Lambda)$ from the data for 42 GRBs ($z > 1.4$) obtained by utilizing the relations calibrated with the sample at $z \leq 1.4$, assuming a Λ CDM model. For a flat universe prior, $\Omega_M = 0.38^{+0.07}_{-0.07}$. (d) The joint confidence regions for $(\Omega_M, \Omega_\Lambda)$ from the 42 GRBs data ($z > 1.4$) obtained by utilizing the relations calibrated with the sample at $z \leq 1.4$, assuming a Riess cosmology. For a flat universe prior, $\Omega_M = 0.39^{+0.07}_{-0.07}$.

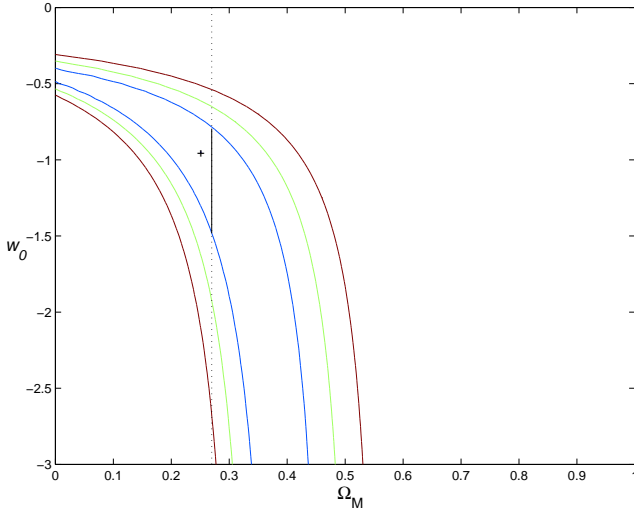


FIG. 3.— Contours of likelihood in the (Ω_M, w_0) plane in the dark energy model with a constant w_0 for a flat universe from the 42 GRBs data ($z > 1.4$) obtained by utilizing the five relations calibrated with the sample at $z \leq 1.4$ using the cubic interpolation method. The plus sign indicates the best-fit values ($\Omega_M = 0.25, w_0 = -0.95$). The contours correspond to 1, 2, and 3- σ confidence regions. For a prior of $\Omega_M = 0.27$ (vertical line), $w_0 = -1.05^{+0.40}_{-0.27}$ (solid part of vertical line).

from the 42 GRB data ($z > 1.4$) obtained by utilizing the five relations calibrated with the sample at $z \leq 1.4$ using the cubic interpolation method. The best-fit values are $\Omega_M = 0.25, w_0 = -0.95$. For a prior of $\Omega_M = 0.27$, we obtain $w_0 = -1.05^{+0.27}_{-0.40}$, which is consistent with the cosmological constant in a 1- σ confidence region.

4. SUMMARY AND DISCUSSION

With the basic assumption that objects at the same redshift should have the same luminosity distance, we can obtain the distance modulus of a GRB at given redshift by interpolating from the Hubble diagram of SNe Ia at $z \leq 1.4$. Since the distance modulus of SN Ia is completely cosmological model independent, the GRB luminosity relations can be calibrated in a completely cosmology independent way. Instead of a hybrid sample in the whole redshift range of GRBs used in most previous treatment, we choose the GRB sample only in the redshift range of SNe Ia to calibrate the relations. We find there are not significant differences between the calibration results obtained by using our interpolation methods and those calibrated by assuming one particular cosmological model (the Λ CDM model or the Riess cosmology) with the same sample at $z \leq 1.4$. This is not surprising, since the two cosmological models are consistent with the data of SNe Ia. Therefore, the GRB luminosity relations calibrated from the cosmological models should not be far from the true ones. However, we stress again that the luminosity relations we obtained here are completely cosmological model independent.

In order to constrain the cosmological parameters, we have applied the calibrated relations to GRB data at high redshift. Since our method does not depend on a particular cosmological model, when we calibrate the parameters of GRB luminosity relations, the so-called circularity problem can be completely avoided. We construct the GRB Hubble diagram and constrain cosmological parameters by the minimum χ^2 method as in SN Ia cosmology. From the 42 GRBs data ($z > 1.4$) obtained by our interpolation method, we obtain $\Omega_M = 0.25^{+0.04}_{-0.05}$ and $\Omega_\Lambda = 0.75^{+0.05}_{-0.04}$ for the flat Λ CDM model, and for the dark energy model with a constant equation of state $w_0 = -1.05^{+0.27}_{-0.40}$ for a flat universe, which is consistent with the concordance model within the statistical error. Our result suggests the concordance model ($w_0 = -1, \Omega_M = 0.27, \Omega_\Lambda = 0.73$) which is mainly derived from observations of SNe Ia at lower redshift is still consistent with the GRB data at higher redshift up to $z = 6.6$.

For the calibration of SNe Ia, the luminosity relations could evolve with redshift, in such a way that local calibrations could introduce biases (Astier et al., 2006; Schaefer, 2007). However, Riess et al. (2007) failed to reveal the direct evidence for SN Ia evolution from analysis of the $z > 1$ sample-averaged spectrum. It is possible that some unknown biases of SN Ia luminosity relations may propagate into the interpolation results and thus the calibration results of GRB relations. Nevertheless, the distance moduli of SN Ia are obtained directly from observations, and therefore the observable distance moduli of SNe Ia used in our interpolation method to calibrate the relations of GRBs are completely independent of any given cosmological model.

Recently, there have been discussions of possible evolution effects and selection bias in GRB relations. Li (2007) used the Amati relation as an example to test the possible cosmic evolution of GRBs and found that the fitting parameters of the relation vary systematically and significantly with the mean redshift of the GRBs. However, Ghirlanda et al. (2008) found no sign of evolution with redshift of the Amati relation and the instrumental selection effects do not dominate for GRBs detected before the launch of the *Swift* satellite. Massaro et al. (2008) investigated the cosmological relation between the GRB energy index and the redshift, and presented a statistical analysis of the Amati relation search-

ing for possible functional biases. Tsutsui et al. (2008) reported a redshift dependence of the lag-luminosity relation in 565 BASTE GRBs. Oguri & Takahashi (2006) and Schaefer (2007) discussed the gravitational lensing and Malmquist biases of GRBs and found that the biases are small. Butler et al. (2007) claimed that the best-fit of the Amati relation presented in the *Swift* sample is inconsistent with the best-fit pre-*Swift* relation at $> 5\sigma$ significance. Butler et al. (2008) showed that the Ghirlanda relation is effectively independent of the GRB redshifts. Nevertheless, further examinations should be required for considering GRBs as standard candles to cosmological use.

Because of the small GRB sample used to constrain the cosmology, the present method provides no more accurate cosmological parameter constraints than the previous works where the GRB and SN Ia samples were used jointly. For example, Wang, Dai, & Zhu (2007) presented constraints on the cosmological parameters by combining the 69 GRB data with the other cosmological probes to get $\Omega_M = 0.27 \pm 0.02$. How-

ever, our main point is not on improvement of the “nominal statistical” error; rather we emphasize that our method avoids the circularity problem more clearly than previous cosmology dependent calibration methods. Therefore, our results for these GRB relations are less dependent on prior cosmological models.

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